令 和 8 年 度

久留米大学大学院医学研究科修士課程 前期入学試験解答

総合生命科学・バイオ統計学専攻 バイオ統計学群(英語)

1. Suppose that the maximum value of function $f(x) = -2x^2 + 8x + k\ (1 \leq x \leq 5)$ is 4.

   (a) Find the value of the constant k.
   (b) Find the minimum value of the function $f(x)$.
   (c) Calculate the area bounded by the parabola $y = f(x)$ and the line $y = 2$.

(a) $f(x) = -2(x^2 - 4x) + k = -2(x-2)^2 + k + 8$　より

$1 \leq x \leq 5$ における$f(x)$の最大値は$f(2) = k + 8$

$k + 8 = 4$ より $k = -4$

(b) $k = -4$ より $f(x) = -2x^2 + 8x - 4$

$1 \leq x \leq 5$ における$f(x)$の最小値は

$$f(5) = -2 \times 5^2 + 8 \times 5 - 4 = -50 + 40 - 4 = -14$$

(c)$y = f(x)$と$y = 2$で囲まれた領域について、xの範囲を求める。

$$-2x^2 + 8x - 4 = 2$$

整理して　$x^2 - 4x + 3 = 0$　　xについて解くと　$x = 1, 3$

よって求める面積は

$$S = \int_1^3 (-2x^2 + 8x - 6)dx = \left[-\frac{2}{3}x^3 + 4x^2 - 6x\right]_1^3 = -\frac{2}{3}(3^3 - 1) + 4(3^2 - 1) - 6(3 - 1)$$

$$= -\frac{2}{3} \times 26 + 4 \times 8 - 6 \times 2 = \frac{8}{3}$$

2. Two players A and B, will play a tennis match. The one who wins 3 sets first will be the winner of the match. Assuming that the probability of A winning a set over B is 2/3, find the probability for each of the following events.

   (a) A wins the match in the 3rd set.
   (b) A wins the match in the 4th set.
   (c) A wins.

(a) $\left(\frac{2}{3}\right)^2 = \frac{8}{27}$

(b) ${}_3C_1 \left(\frac{2}{3}\right)^2 \cdot \frac{1}{3} \cdot \frac{2}{3} = \frac{8}{27}$

(c) The probability that A wins in the 5th set is ${}_4C_3 \left(\frac{2}{3}\right)^2 \cdot \frac{2}{3} = \frac{16}{81}$.

Thus, the probability that A wins is $\frac{8}{27} + \frac{8}{27} + \frac{16}{81} = \frac{64}{81}$.

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3. In a math class, the exam results show an average score of 42 points and a variance of 18.
   (a) If 30 points are added to each student's score, what will be the new average score and variance for the class?
   (b) If each student's score is doubled, what will be the new average score and variance for the class?
   (c) If we subtract the class average (42) from each student's score and then ~~divide~~ the result is divided by the square root of the variance ($\sqrt{18}$), what will be the new average score and variance?

[Note] Assuming the students' scores are $x_1, \ldots, x_n$, then the variance is defined as $\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$ where $\bar{x}=\frac{1}{n}\sum_{i=1}^{n}x_i$ is the average score.

(a) The average increases by 30 points, and the variance does not change (18).
(b) The average is 42*2=84, and the variance is 18*2^2=72.
(c) The average is 0 and the variance is 1.

4. Solve the following problems.
   (a) Find the value.
$$\sqrt[3]{3}\times\sqrt[3]{243}$$
   (b) Find the value.
$$(\log_3 125+\log_9 5)(\log_{25} 3+\log_5 9)$$
   (c) Solve the following equation.
$$2+\log_5 x=(\log_5 x)^2$$

(a)
$$\sqrt[3]{3}\times\sqrt[3]{243}=3^{\frac{1}{3}}\times3^{\frac{5}{3}}=3^2=9$$

(b)
$$(\log_3 125+\log_9 5)(\log_{25} 3+\log_5 9)=\left(\log_3 5^3+\frac{\log_3 5}{\log_3 9}\right)\left(\frac{\log_5 3}{\log_5 25}+\log_5 3^2\right)$$
$$=\left(3\log_3 5+\frac{\log_3 5}{2}\right)\left(\frac{\log_5 3}{2}+2\log_5 3\right)$$
$$=\frac{7}{2}\log_3 5\times\frac{5\log_3 3}{2\log_3 5}=\frac{35}{4}$$

(c)
対数の真数条件より $x>0$
$\log_5 x=t$ とおくと、$2+t=t^2$より $t^2-t-2=0$
$$(t-2)(t+1)=0$$
$$t=-1,2$$
$t=-1$のとき、$\log_5 x=-1$ より$x=5^{-1}=\frac{1}{5}$
$t=2$のとき、$\log_5 x=2$ より$x=5^2=25$
これらは $x>0$ を満たす。
よって $x=\frac{1}{5},25$